LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – NOVEMBER 2015

MT 1818 - DIFFERENTIAL GEOMETRY

Date : 11/11/2015 Dept. No. Max. : 100 Marks

Time : 01:00-04:00

Answer ALL the Questions:

1. a) Find the curvature and torsion of the curve $\vec{x} = (u, u^2, u^3)$. **(5)**

OR

b) For the curve $\vec{x} = (e^{-u}sinu, e^{-u}cosu, e^{-u})$. Find at any point u of the curve (i) unit tangent (ii) equation of the tangent (iii) equation of the normal plane. **(5)**

c) (i) Find the equation of the osculating plane at a point on the curve of the intersection of the cylinders $x^2 + z^2 = a^2$, $y^2 + z^2 = b^2$.

(ii) Show that the tangent at a point of the curve of the intersection of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ and the confocal whose parameter is λ is given by $\frac{x(X-x)}{a^2(b^2-c^2)(a^2-\lambda)} = \frac{y(Y-y)}{b^2(c^2-a^2)(b^2-\lambda)} = \frac{z(Z-z)}{c^2(a^2-b^2)(c^2-\lambda)}$. **(9 +6)**

OR

d) (i) State and prove Serret-Frenet formula.

(ii) Use Serret-Frenet formula, to find an expression for curvature. **(10 + 5)**

2. a) Find the lines that have four point contact at (0, 0, 1) with the surface

$$x^4 + 3xyz + x^2 - y^2 - z^2 + 2yz - 3xy - 2y + 2z = 1.$$ **(5)**

OR

b) Find the necessary and sufficient condition that a curve to be a helix. **(5)**

c) Find the equations of the curve whose curvature and torsion are constants. **(15)**

OR

d) Derive the equation of evolute of a curve. Also find the curvature and torsion of an evolute. **(15)**

3. a) Show that the envelope of the plane that forms with the coordinate planes a tetrahedron of constant volume. **(5)**

OR

b) Give the quadratic form of first fundamental form. Also calculate the fundamental magnitudes for the surface of revolution. **(5)**

c) Prove that the necessary and sufficient condition for the surface may be developable is that its Gaussian surface is zero. **(15)**

OR

d) Find the edge of regression of the developable surface that passes through the parabolas $z^2 = 4ay, x = 0; y^2 = 4az, x = a$. **(15)**

4. a) State and prove Meusnier's theorem. **(5)**

OR

b) Prove that the ratio of the second fundamental form to the first fundamental form is the normal curvature of the surface. **(5)**

c) (i) Show that the Dupin indicatrix at every point of the right helicoids is a rectangular hyperbola.

(ii) Find the principal curvature of the coincoid $x = u\cos\theta, y = u\sin\theta, z = f(\theta)$.

(6 + 9)

OR

d) (i) Define geodesic on a surface. Prove that the curves $u + v =$ constant are geodesic on a surface with metric $(1 + u^2)du^2 - 2uvdudv + (1 + u^2)dv^2$.

(ii) Find the differential equation of lines of curvature through a point on the surface $z = f(x, y)$. **(9 + 6)**

5. a) Prove that the Gaussian curvature of a surface is a bending invariant. **(5)**

OR

b) Derive Weingarten's equations. **(5)**

c) Derive the equations of Gauss. **(15)**

OR

d) (i) Prove that the sphere is the only surface in which all points are umbilics.

(ii) If κ_1 and κ_2 are the principal normal curvatures, derive Codazzi equations.

(8 + 7)

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